Smooth Local Panel Projections

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In this note, I extend smooth local projections to panel data and use Monte Carlo methods to explore the bias and variance properties of smooth local panel projections (SLPP). SLPP allows researchers to penalize the impulse respond toward a polynomial, while standard local panel projections (PLP) are nonparametric but result in theoretically unappealing IRFs because they are too lumpy. In general, an econometrician should prefer SLPP over standard local panel projections unless he places a strong weight on bias or the sample size is very large. When the true impulse response function (IRF) is linear, the econometrician should always use SLPP. Additionally, I show that a wild cluster bootstrap is robust and preferable to regular clustered standard errors unless the true impulse response function is very lumpy. Finally, I apply SLPP to oil news shocks from Arezki, Ramey, and Sheng (2017).

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1 Introduction

Economists increasingly rely on panel local projections to study the dynamic effects of shocks. Because of the extra dimension of variation offered by panel data over time series, Herbst and Johannsen (2024) document that a large share of studies utilizing local projections rely on panel data. Indeed, with the advent of panel local projections in difference-in-difference studies, panel local projections are an important tool for both macroeconomists and microeconomists alike (Dube et al. 2023). At the same time, because panel local projections are nonparametric, the resulting impulse responses are unappealingly lumpy; coefficients jump from horizon to horizon in a way that is a result of noise and cannot be meaningfully justified with economic theory. In this note, I make several contributions toward solving that problem. First, I extend smooth local panel projections from time series to panel data and offer a flexible way to compute confidence intervals. Second, I give evidence from Monte Carlo simulations that as long as researchers have some confidence in the shape of the impulse response, they should choose smooth local panel projections (SLPP) over standard local panel projections. Third, I apply the method to studying the dynamic responses of macroeconomic variables to oil news shocks. Fourth, I provide an easy to use R package and vignette.[1]

SLPP offers researchers the ability to discipline the behavior of the impulse response function with panel data. Theory typically suggests that impulse responses should be linear, quadratic, or if overshooting is a possibility, a cubic function. SLPP allows researchers to penalize impulse responses toward one of those polynomials by linking impulse response coefficients across horizons using B-spline basis functions. This is not a new advance over previous work; Barnichon and Brownlees (2019) develop this methodology for time series data. Rather, the key benefit is instead the new ability to leverage both the cross-sectional and time series dimensions to appropriately select the penalty parameter and appeal to researchers using both micro and macro data. Additionally, panel data offer a rich variety of options for standard errors. As Barnichon and Brownlees (2019) point out, it is unclear how to construct standard errors for penalized IRFs. This paper relies on a wild bootstrap that can cluster at any level of aggregation, but can be extended to other options.

Additionally, in keeping with Li, Plagborg-Møller, and Wolf (2021), I show that researchers should generally prefer SLPP over standard panel LP (PLP). When the shape of the true IRF is linear, econometricians should always prefer SLPP. This is in keeping with

1. The package, called `slpp` and vignette can be found here. `slpp` relies on the `fixest` package and uses data on oil shocks from Arezki, Ramey, and Sheng (2017) to demonstrate the package's capabilities.

Li, Plagborg-Møller, and Wolf (2021), which shows in a time series context that standard LP exhibit so much variance that one should simply use penalized LP. The same is true in this context, especially when the researcher has a prior on the shape of the impulse response function. I also show that the wild cluster bootstrap has good properties, particularly if the true IRF is not too lumpy, whereas the confidence intervals from standard confidence intervals are typically too wide in smaller samples. This result is similar to the analysis of Kilian and Kim (2011), which also shows that the LP estimator has poor length properties in time series. I do not analyze SLPP with instrumental variables, but it is easy to implement.

Roadmap. Section 2 describes the SLPP estimator and Section 3 discusses the results of Monte Carlo simulations. Section 4 applies the method to oil news shocks. Section 5 concludes.

2 Smooth Local Panel Projections

In this section, I outline the procedure for estimating smooth local projections for panel data. If readers are already familiar with Barnichon and Brownlees (2019), they can proceed to the following section; much of the following simply rehashes their Section 2, but suitably adapted for panel data. Consider a typical dynamic panel regression

$$y_{i,t+h} = G(i,t) + \beta_h x_{i,t} + \text{Controls} + \nu_{i,t+h},$$

for cross-sectional units $i = 1, \ldots, N$ estimated at time $t + h$ given a shock to $x_{i,t}$ at time t. The sequence of coefficients β_h from t to $t + h$ consist of the impulse response up to h horizons out. In general, our interest is in smoothing the coefficient B_h across horizons. $G(i,t)$ is a set of additive fixed effects that depend on unit i at time t but are not necessarily two-way time and individual fixed effects. For example, one may wish to include an industry fixed effect or an interaction between time and industry when studying how firms react to shocks. All that matters in this context is that the dependent and independent variables are demeaned with respect to $G(i,t)$. Controls may include anything else the researcher wishes.[2]

As in Barnichon and Brownlees (2019), the goal is to make the coefficient β_h a smooth function of the impulse horizon. To do that, we simply use a B-spline basis function to

2. If y is nonstationary, the researcher would should include lags of y.Montiel Olea and Plagborg-Møller (2021) show that nonstationary LPs are consistent in time series as long as they include lagged regressors. Of course, this also makes them subject to Nickell bias in a panel data setting.

approximate the coefficient

$$\beta_h \approx \sum_{k=1}^{K} b_k B_k(h)$$

for K sufficiently large and where $B_k : \mathbb{R} \to \mathbb{R}$. The B-splines link coefficients across horizons. The basis functions are polynomial pieces of order q which are continuous at the kinks by imposing the constraint that all derivatives up to order $q-1$ are continuous at the kinks. There are $q+2$ kinks and each one is called an inner knot. See Barnichon and Brownlees (2019) and references therein for a more detailed discussion of basis functions. Following their discussion, I use a cubic basis function throughout.

To set notation, let variables with a tilde denote their demeaned version with respect to fixed effects $G(i,t)$. Let H_{max} denote the maximum forecast horizon. To set notation, let $\boldsymbol{y}_{i,t}$ denote the vector of outcomes $(\widetilde{y}_{i,t}, \ldots, \widetilde{y}_{\{i,\min\{T,t+H_{\max}\}})'$ with length d_t. Let $\boldsymbol{x}_{i,t}$ for $t = 1, \ldots, T$ denote the $d_t \times K$ matrix with element (h, K) equal to $B_k(h)x_{i,t}$. Next, let $\mathcal{Y}$ denote the stacked vector individual vectors $y_{i,t}$ and $\mathcal{X}$ denote the stacked matrices for individuals $\boldsymbol{x}_{i,t}$. Crucially, both $\mathcal{Y}$ and $\mathcal{X}$ must retain the original order with respect to the original order of individual units. Finally, let θ denote the vector of B-splines coefficients $(b_1, \ldots, b_K)$.

In principle, one could smooth IRFs for multiple variables in SLPP. But in practice, it is hard to think of an application where that would be either practical or relevant because researchers frequently only care about the response of a single outcome variable to a single exogenous shock. Consequently, I focus solely on cases where one wants the smoothed IRF for one variable. To account for the case with controls, suppose there is a vector of controls $\boldsymbol{C}_{i,t}$ of length J. With $J = 1$, we would have a $d_t \times K$ matrix with element (h, K) equal to $c_{i,t}$. That is, rather than multiply by the basis function, we simply multiply by the identity matrix. The same procedure can be extended for multiple controls.

With that notation, a cookbook for estimating SLPP is the following.

1. **Demean data with respect to fixed effects** $G(i,t)$**.**

2. **Construct matrices** $\mathcal{Y}$ **and** $\mathcal{X}$**.** Note that maintaining order is crucial for the demeaned data. In particular, demeaned data must be ordered within individual clusters by time and horizon regardless of fixed effects. After this step, what follows is essentially the same as in Barnichon and Brownlees (2019).

3. **Select the polynomial order of the impulse response function.** In the following step, I discuss how one can use a ridge regression to penalize the impulse response toward a polynormial of order $r-1$.

4. **Estimate ridge regression**:

$$\begin{aligned}\hat{\theta} &= \arg\min_{\theta}\{\|\mathcal{Y} - \mathcal{X}\theta|^2 + \lambda\theta' \boldsymbol{P}\theta\} \\ &= \left(\mathcal{X}'\mathcal{X} + \lambda \boldsymbol{P}\right)^{-1} \mathcal{X}'\mathcal{Y},\end{aligned} \tag{1}$$

   where $\lambda > 0$ is a shrinkage parameter and P is a symmetric positive semidefinite penalty matrix. λ determines the bias/variance trade-off in shrinking the impulse response toward the chosen polynomial order. Researchers should estimate (1) over a grid of penalty parameters. It is common in typical ridge regression analyses to use a logarithmic grid ranging from 10^{-3} to 10^3.

5. **Use generalized cross-validation to select λ.** λ penalizes the IRF toward a polynomial of order $r-1$. Because panel data can be computationally expensive to deal with, I propose using a version of generalized cross-validation (Golub, Heath, and Wahba 1979) rather than cross-fold validation. This relies on the formula

$$GCV = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{y_i - \hat{y}_i}{1 - \frac{\log(\text{trace}(\boldsymbol{S}))}{N}}\right]^2,$$

   where $\hat{y}_i$ are predicted values, N is the number of observations, and $\boldsymbol{S}$ is the smoother matrix given by:

$$S = \mathcal{X}(\mathcal{X}'\mathcal{X} + \lambda\boldsymbol{P})^{-1}\mathcal{X}'.$$

   Because trace($\boldsymbol{S}$) is computationally expensive, I estimate the trace with a stochastic vector approach (Hutchinson 1990). The trace of $\boldsymbol{S}$ is defined as the sum of its diagonal elements, $\text{trace}(\boldsymbol{S}) = \sum_i S_{ii}$. This can be stochastically estimated by following three steps:

   (a) Generating Random Vectors: Generate random vectors z where each component of z is independently drawn from a standard normal distribution.

   (b) Compute the matrix-vector product $\boldsymbol{S}z$.

   (c) Estimate the trace with

$$\text{trace}(\boldsymbol{S}) \approx \mathbb{E}[(z'\boldsymbol{S}z)].$$

   Then, for a grid of candidate penalty parameters, I select the one which minimizes the GCV. This is in contrast to Barnichon and Brownlees (2019), which uses K-fold cross-validation. That may be a better approach here, but it is computationally too expensive for many practical applications.

6. **Construct confidence bands.** In a time series context, Barnichon and Brownlees (2019) propose using Newey-West standard errors. Because panel data researchers are often interesting in clustering standard errors, I propose instead using a wild cluster bootstrap. Here, we can cluster around a chosen characteristic, which allows a great deal of flexibility. At the same time, it can be somewhat more time-consuming for research designs using instrumental variables.

The method can be readily extended to instrumental variables as in, for example, Jordà, Schularick, and Taylor (2020). Suppose there is some matrix of demeaned and appropriately stacked instruments $\mathcal{Z}$ for $\mathcal{X}$. Then the SLPP-IV estimator is

$$\hat{\theta} = \left(\mathcal{Z}'\mathcal{X} + \lambda \boldsymbol{P}\right)^{-1} \mathcal{Z}'\mathcal{Y}. \tag{2}$$

The rest of the procedure is largely the same as discussed above, except that the wild bootstrap is applied to both the first stage and the second stage.

3 SLPP in Practice

In this section, I proceed in two steps to discuss how researchers should implement SLPP. First, I conduct Monte Carlo simulations to compare SLPP to standard panel local projections (PLP). Second, I discuss how researchers should proceed if they are unsure about the shape of the true impulse response function.

3.1 Monte Carlo Simulations

To evaluate the performance of SLPP vis-à-vis standard PLP, I consider simple regressions of the form

$$y_{i,t+h} = \alpha_i + T_t + \beta_h x_{i,t} + \varepsilon_{i,t}, \tag{3}$$

where α_i is an individual fixed effect, T_t is a time fixed effect, and $x_{i,t}$ is the time$-t$ shock to y. This is perhaps the most commonly used regression in practice. I evaluate SLPP under three different DGPs, where the shape of the IRF is linear in the first, quadratic in the second, and cubic in the third. These are the most theoretically appealing shapes. Figure 1 plots the targeted impulse responses.

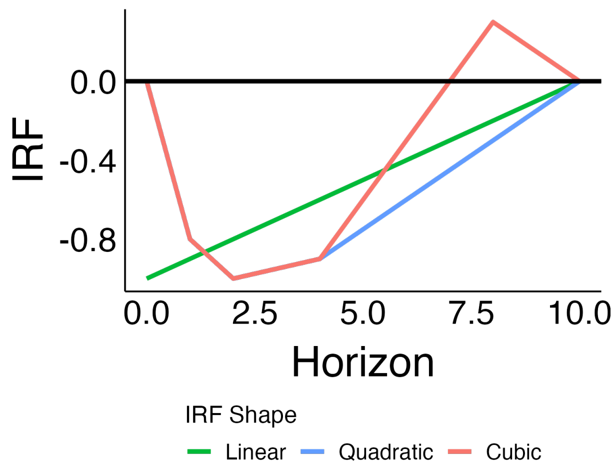


Figure 1: True IRFs used for Monte Carlo simulations.

The Monte Carlo simulations test a variety of panel sizes. For each panel size, I generate 5000 replications of sample data for each IRF type. To test the performance of the bootstrap, I draw 5000 bootstrap samples. Throughout, I use a balanced panel. Each replication selects a penalty parameter from a grid of penalty parameters from 10^{-3} to 10^3 distributed logarithmically over 100 grid points.

Point Estimates

To start, suppose we know what the shape of the underlying process is. In general, this will not be true and I will address the contrary case in the following subsection. Following Li, Plagborg-Møller, and Wolf (2021), I compare the performance of SLPP and PLP at each horizon using the following loss function

$$\mathcal{L}_h\left(\hat{\beta}_h, \beta_h, \omega\right) = \omega\left(\mathbb{E}\left[\hat{\beta}_h - \beta_h\right]\right)^2 + (1-\omega)\mathbb{V}\left[\hat{\beta}_h\right], \tag{4}$$

where $\omega \in [0,1]$ is the weight the econometrician places on bias. To place a summary number on the comparison, I compute the discounted sum of losses at each horizon as

$$\mathbb{L}^i(\omega) = \sum_{h=0}^{H}\left(\frac{1}{1+r}\right)^h \mathcal{L}_h,$$

for $i \in \{\text{SLPP}, \text{PLP}\}$ where $r \geq 0$ is the econometrician's discount rate and PLP refers to standard panel local projections. r may be greater than zero because one cares more about minimizing losses at nearer horizons. I set $r = 0$. Finally, I compare the performance of

SLPP and PLP by taking the ratio

$$\nu(\omega) = \frac{\mathbb{L}^{\text{PLP}}(\omega)}{\mathbb{L}^{\text{SLPP}}(\omega)}.$$

If $v(\omega) > 1$, then the econometrician should prefer SLPP to PLP. Conversely, if $v(\omega) < 1$, then the econometrician should prefer PLP. In Figure 2, I plot the curve $v(\omega)$ for $N = 400$ individuals and $T = 100$ time periods each IRF type. Although the sample is large, which means that the variance of the PLP estimator is substantially diminished, an econometrician would prefer SLPP for both cubic and quadratic IRFs unless the weight placed on bias is close to 0.9. In other words, SLPP substantially outperforms PLP. Moreover, an econometrician would never prefer PLP over SLPP if the true IRF is linear.

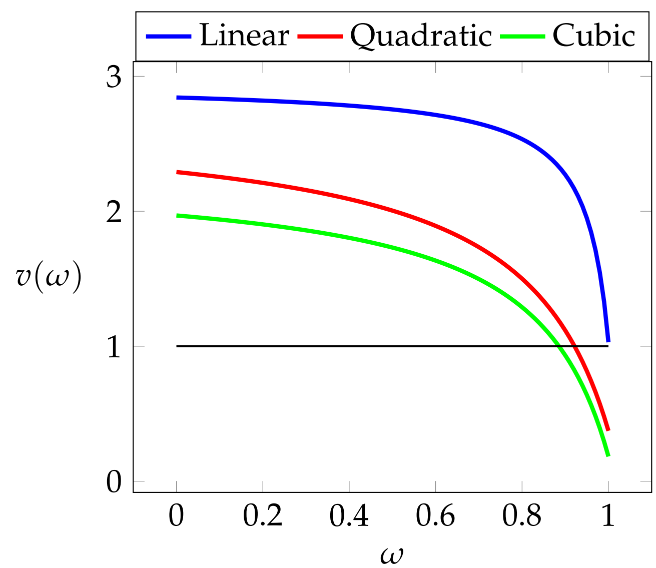


Figure 2: $v(\omega)$ curve for $T = 100$ time periods and $N = 400$ individuals.

As a summary measure, consider the threshold variable $\bar{\omega}$, where we say that $\omega > \bar{\omega}$ means that the researcher should choose SLPP. Table 1 tabulates $\bar{\omega}$ for linear, quadratic, and cubic IRFs for sample sizes which are empirically relevant in the data. If the true IRF is linear, then the researcher should never choose PLP even if he has an overwhelming prefernce for bias. As more curvature enters the IRF through either a quadratic or a cubic IRF, then the dominance of SLPP recedes. Even so, threshold values around 0.9 indicate researchers should generally prefer SLPP. Table 4 tabulates the value of the loss function for $\omega \in \{0, 0.5, 1\}$ to give an idea of the shape of the $v(\omega)$ function for varying IRF shapes and sample sizes.

N	T	$\bar{\omega}$ Linear	Quadratic	Cubic
50	25	1.00	0.97	0.97
100	25	0.99	0.95	0.95
250	25	0.99	0.89	0.90
50	50	0.99	0.97	0.97
100	50	0.99	0.94	0.95
250	50	0.99	0.87	0.88
50	100	0.99	0.96	0.96
100	100	0.99	0.92	0.94
250	100	0.99	0.83	0.86

Table 1: Threshold value $\bar{\omega}$ for different underlying IRFs and varying sample sizes. If the preference for bias exceeds $\bar{\omega}$, then the econometrician should prefer PLP.

Confidence Interval Performance

Standard confidence intervals generally cannot be used with ridge regression, so I instead construct wild cluster bootstrap intervals. In this subsection, I compare the PLP confidence intervals created from clustering at the individual level to a wild cluster bootstrap at the same level of aggregation. Table 2 documents coverage properties of the wild cluster bootstrap for SLPP compared to clustered standard errors constructed analytically for PLP. Each column reports the percent of the time the 95% confidence interval contains the true parameter. This is value averages over all horizons. Across all sample sizes, the PLP and SLPP confidence intervals perform nearly identically in the linear case. However, introducing some curvature in the true IRF substantially affects the performance of the wild cluster bootstrap when the time dimension is short. When the time dimension is at least fifty, then it performs similarly to the analytically computed cluster standard error for PLP, but a short time dimension is especially problematic for the bootstrap. It is not surprising that the bootstrap struggles in small samples when there is a lot of curvature. SLPP is biased by construction and will tend to underestimate the curvature of the function. For example, if there is a theoretical minimum at $h = 3$, then the $\hat{\beta}_3 < \beta_3$ because it is biased upward by the smoothing with the higher points. One can get around this by using more knots or increasing the polynomial order, but the trade-off is that the estimator just approaches the PLP estimator.

N	T	Linear		Quadratic		Cubic	
		PLP	SLPP	PLP	SLPP	PLP	SLPP
50	25	1.922	0.878	1.941	1.080	1.934	1.216
100	25	0.864	0.402	0.864	0.492	0.864	0.551
250	25	0.683	0.316	0.684	0.391	0.684	0.438
50	50	1.913	0.855	1.932	1.067	1.925	1.206
100	50	0.859	0.389	0.861	0.484	0.857	0.547
250	50	0.680	0.308	0.679	0.384	0.679	0.435
50	100	1.919	0.819	1.911	1.028	1.911	1.166
100	100	0.858	0.373	0.860	0.469	0.856	0.536
250	100	0.680	0.296	0.679	0.371	0.678	0.427

Table 2: Coverage properties of PLP standard errors versus the wild cluster bootstrap.

The length properties of the wild cluster bootstrap for SLPP are also be quite promising for applied researchers. Table 3 tabulates the length of the 95% confidence interval for different sample sizes and IRF types. In smaller samples, the length of the SLPP confidence interval is about half that of the equivalently clustered confidence interval and remains significantly smaller even in large samples. Because the coverage properties are nearly as good for the SLPP bootstrap, this means that the SLPP is much better at correctly determining significance as long as there is not too much curvature in the true IRF and the sample size is not too small.

N	T	Linear		Quadratic		Cubic	
		PLP	SLPP	PLP	SLPP	PLP	SLPP
50	25	0.936	0.922	0.931	0.892	0.931	0.930
100	25	0.929	0.877	0.903	0.788	0.931	0.896
250	25	0.915	0.811	0.874	0.745	0.925	0.870
50	50	0.937	0.938	0.939	0.913	0.934	0.932
100	50	0.942	0.937	0.941	0.851	0.950	0.923
250	50	0.942	0.933	0.924	0.801	0.945	0.912
50	100	0.937	0.951	0.939	0.909	0.937	0.921
100	100	0.945	0.945	0.947	0.837	0.953	0.929
250	100	0.949	0.949	0.943	0.784	0.948	0.921

Table 3: Length properties of PLP standard errors versus the wild cluster bootstrap.

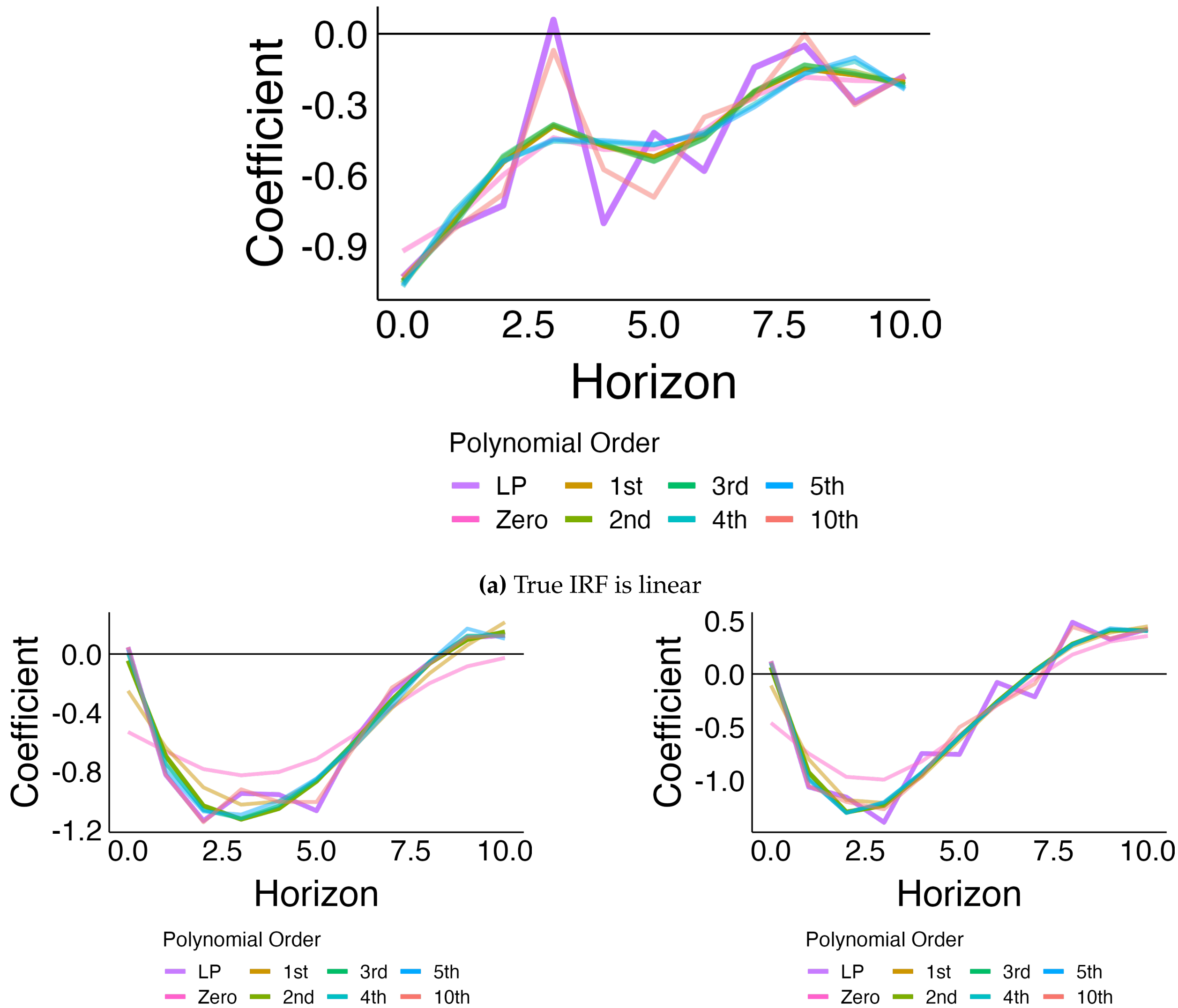
3.2 Uncertainty about the IRF

Economists often have in mind the shape of an IRF prior to engaging with the data. Typically, a model or a class of models maps into a linear, quadratic, or cubic IRF. However, one may wish to use the IRF to distinguish between models. For example, capital adjustment costs yield approximately linear IRFs, while investment adjustment costs produce hump-shaped IRFs. In such situations, when the econometrician has a weak prior on the shape of the IRF, standard LP will be practically useless because it is too noisy to determine with any certainty the shape of the IRF. In contrast, a tool like SLPP can be quite useful. Consider a simple panel local projection of the form

$$y_{i,t+h} = \alpha_i + T_t + \beta_h x_{i,t} + \varepsilon_{i,t+h}.$$

Since a significant part of the procedure for SLPP is selecting a polynomial order to discipline the IRF, one may worry that about naively choosing the wrong polynomial order will lead to incorrect results. For example, one may select a linear polynomial when the true IRF is quadratic. However, SLPP is robust to this concern. Through appropriate selection of the penalty parameter, the resulting IRFs will be quite similar for lower polynomial orders. To show that, I generate sample data with $N = 250$ cross-sectional units and $T = 50$ time periods. In Figure 3, I plot the impulse responses for several different polynomial orders when the true DGP is either linear, quadratic, or cubic. It turns out that generalized cross-validation appropriately selects a penalty parameter for all three cases such that it does not matter very much in practice whether the selected polynomial order is less than four. However, as the polynomial order increases, the SLPP estimator increasingly resembles the PLP estimator.

The practical takeaway is that it will generally lead to robust results if the econometrician selects a polynomial of order one, two, or three because the penalty selection process will ensure that they are all similar. However, one should not select a polynomial order much higher than three and there is not a good econometric reason to do that anyway. Moreover, unless one thinks that the IRF is a jump process with a unit root, then it likewise is not recommended to select a polynomial of order zero.



(a) True IRF is linear

(b) True IRF is quadratic

(c) True IRF is cubic

Figure 3: Impulse responses for different polynomial orders when the underlying true IRF is linear, quadratic, or cubic. Sample data are generated from $N = 250$ cross-sectional units and $T = 50$ time periods.

4 An Application to the Dynamic Effects of Oil Shocks

Arezki, Ramey, and Sheng (2017) provides an ideal setting to demonstrate the utility of SLPP. That paper studies the dynamic effects of oil news shocks using a cross-country panel. They use a variety of methods to study this including PLP. However, they note in the appendix that, compared to the dynamic panel estimator they employ,"the estimated impulse responses [from PLP] are more erratic and often less precise." Many use cases

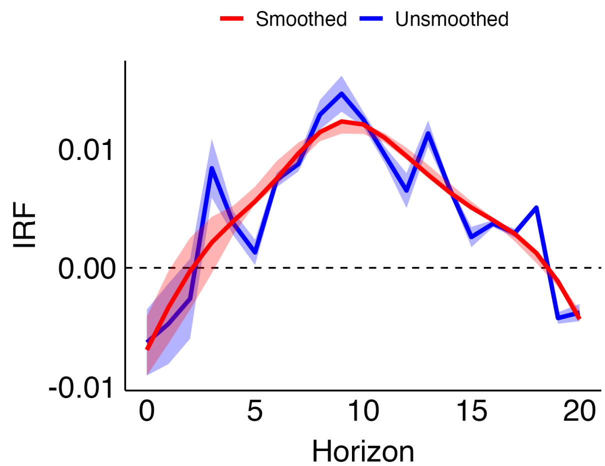


Figure 4: Response of current account/GDP for the PLP in blue and the SLPP (in red) with a 95% confidence interval and 1000 bootstrap replications.

involve similar sample sizes and consequently exhibit similarly lumpy IRFs, so this paper is an excellent illustration.

The key contribution of Arezki, Ramey, and Sheng (2017) is to construct, using a novel data source, a panel of oil news shocks and show that the impulse responses of macro variables to these shocks conforms to a simple open economy model. The paper also shows theoretical IRFs, which means it is straightforward to discipline the polynomial order of the impulse response. In my illustration of SLPP, I use their baseline measure of the oil news shock, which is the net present value of oil discovery scaled by time t GDP and discounts the future using country-specific discount rates. As a first application, consider the response of the current account scaled by GDP to an oil shock in their full sample of data.[3] Since the current account is stationary, this comes from the regression

$$y_{i,t+h} = \alpha_i + T_t + \beta_h x_{i,t} + \varepsilon_{i,t+h}, \tag{5}$$

where α_i is a country i fixed effect and T_t is a time fixed effect. $x_{i,t}$ is the oil news shock for country i. Figure 4 plots the impulse response of the current account for up to twenty years out penalized to a quadratic along with a 95% confidence interval. In blue, I also plot the PLP estimator. The PLP estimator is incredibly lumpy by comparison, although there is very little difference in the confidence interval.

As a second example, consider the response of investment/GDP, plotted in Figure 5. Under KPR preferences, the saving rate should decline, rise, and then fall negative again.

3. I use their replication data and simply filter out any observations without a current account or an oil shock.

This suggests disciplining the IRF to a cubic. The smoothed IRF in Figure 5 supports the theory, but the PLP estimator IRF is ambiguous.

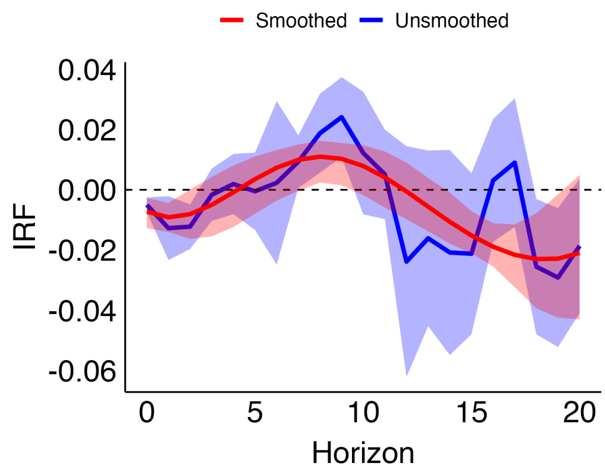


Figure 5: IRF for consumption response for the PLP in blue and the SLPP (in red) with a 95% confidence interval and 1000 bootstrap replications.

Finally, note that SLPP can easily accommodate cumulative IRFs given by

$$y_{i,t+h} - y_{i,t-1} = \alpha_i + T_t + \beta_h x_{i,t} + \text{controls} + \varepsilon_{i,t}. \tag{6}$$

Theory implies that the consumption response to an oil news shock should be flat (Arezki, Ramey, and Sheng 2017, p. 115) to reflect consumption smoothing. Figure 6 plots the consumption response to an oil news shock. As predicted, it is essentially flat and positive, albeit not significantly so.

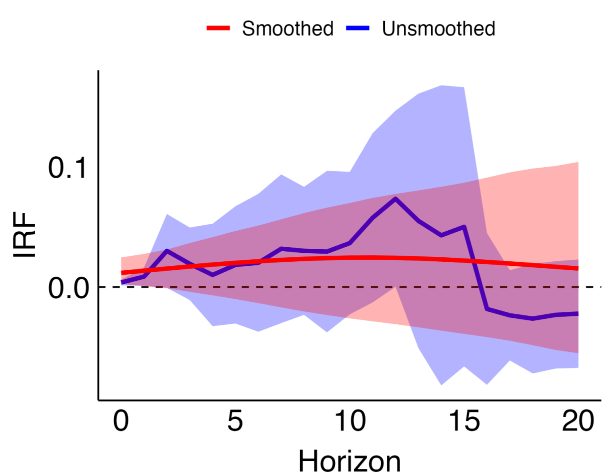


Figure 6: IRF for consumption response for the PLP in blue and the SLPP (in red) with a 95% confidence interval and 1000 bootstrap replications.

5 Concluding Remarks

In this paper, I extend smooth local projections from time series to panel data. I show that the properties of the SLPP estimator are generally robust and SLPP is preferable to standard panel local projections except in rather extreme circumstances. Additionally, I show that the estimator is robust to selecting the wrong polynomial order. Finally, I demonstrated the utility of the estimator through an application to the dynamic effects of tax policy at the firm level. The cumulative effects are large and persistent over time.

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N	T	Linear			Quadratic			Cubic		
		$\omega = 0$	$\omega = 0.5$	$\omega = 1$	$\omega = 0$	$\omega = 0.5$	$\omega = 1$	$\omega = 0$	$\omega = 0.5$	$\omega = 1$
50	25	4.66	4.13	1.00	3.18	2.67	0.84	2.55	2.38	0.67
100	25	4.61	3.68	0.98	3.03	2.34	0.81	2.61	2.29	0.64
250	25	4.85	3.06	0.98	3.00	1.78	0.82	2.51	1.92	0.67
50	50	5.02	4.86	0.93	3.31	3.04	0.39	2.57	2.44	0.34
100	50	4.79	4.52	0.87	3.21	2.69	0.44	2.56	2.33	0.27
250	50	5.20	4.33	0.88	3.19	2.18	0.45	2.48	2.02	0.27
50	100	5.40	5.32	0.50	3.46	3.13	0.14	2.75	2.59	0.12
100	100	5.45	5.25	0.62	3.52	2.88	0.17	2.68	2.41	0.10
250	100	5.42	4.96	0.60	3.47	2.25	0.13	2.67	2.10	0.07

Table 4: $v(\omega)$ for $\omega \in \{0, 0.5, 1\}$.